APPENDIX.

EPHEMERIS

FOR THE YEAR

1837,

FOR THE MERIDIAN

OF THE

ROYAL OBSERVATORY AT GREENWICH.

CONFIGURATIONS OF THE SATELLITES OF JUPITE

At 12^h, Mean Time.

Day of the Month.	*West.*		*East.*
1	4· ·2	O	·3
2	4·	O	·2 ·1 3·
3	4· 1·	O	3· 2·
4	·4 2· 3·	O	·1
5	·4 3· ·1 ·2	O	
6	·4 ·3	O	1· ·2
7	·4 ·1	O	2·
8	.2 1	O	·4 ·3
9		O	·1 ·2 ·4
10	1·	O	2· 3· ·4
11	3: 2:	O	·1 ·4
12	3· ·2 ·1	O	4.
13	·3	O	1· ·2 4·
14	·1 ·3	O	2· 4·
15	2·	O	1· 4· ·3
16	4·	O	·3 ·
17	4· 1·	O	3: 2:
18	4· 3: 2:	O	·1
19	4· 3· ·2 1·	O	
20	·4 ·3	O	1· ·2
21	·4 ·1 ·3	O	2·
22	·4 2·	O	1· ·3
23	·4 ·2 ·1	O	3·
24	1· O ·4	O	2· 3·
25	3· 2·	O	·1 ·4
26	3· ·2 1·	O	·4
27	·3	O	·2 ·1 ·4
28	·3 ·1	O	2· 4.
29	2·	O	1· ·3 4·
30	·2 ·1	O	·3 4·
31	O 1·	O	2· 3· 4·

This Table represents, at 12^h after *Mean Noon* of each day of the month, the relative positio the images of Jupiter and his Satellites, as they would appear (disregarding their latitudes) i inverting telescope. Jupiter is indicated by the white circles (O) in the centre of the the Satellites by points. The numerals 1, 2, 3, and 4, annexed to the points, serve to distin the Satellites from each other; and their positions are such as to indicate the directions of the lites' motions, which are in all cases to be considered as *towards the numerals.* When a Satell at its greatest elongation, the point is placed above or below the centre of the numeral. A circle (O) at the left or right hand of the page, denotes that the Satellite placed by the side o *on* the disc of Jupiter, and a black circle (●) that it is either *behind* the disc, or in the shado Jupiter.

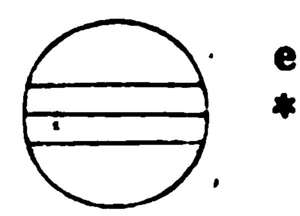
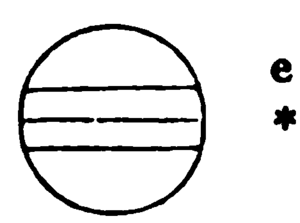
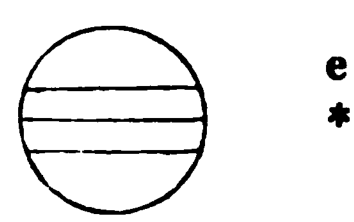
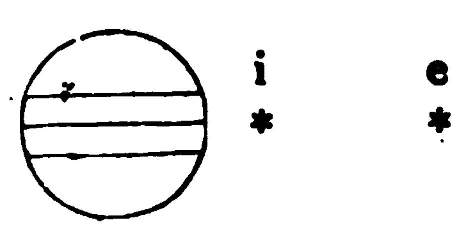
MEAN TIME.

PHASES OF THE MOON.

	d	h	m
● New Moon	4	22	7·8
☽ *First Quarter*	11	21	38·3
○ Full Moon	20	2	23·3
☾ *Last Quarter*	27	17	30·8

	d	h
☾ Perigee	4	4
☾ Apogee	16	19

ECLIPSES OF THE SATELLITES OF JUPITER.

SATELLITE.	Day of the Month.	Mean Time.	Sidereal Time.	PHASE as seen in an inverting Telescope.	
		h m s	h m s		
I.	1	21 52 23 ·6	4 33 18 ·4	Em.	
	3	16 21 4 ·5	23 8 58 ·0	Em.	
	5	10 49 49 ·8	17 44 42 ·0	Em.	
	7	5 18 30 ·6	12 20 21 ·5	Em.	
	8	23 47 15 ·9	6 56 5 ·5	Em.	
	10	18 15 56 ·0	1 31 44 ·2	Em.	
	12	12 44 40 ·6	20 7 27 ·6	Em.	
	14	7 13 20 ·7	14 43 6 ·4	Em.	
	16	1 42 4 ·4	9 18 48 ·8	Em.	
	17	20 10 44 ·0	3 54 27 ·1	Em.	
	19	14 39 28 ·0	22 30 9 ·7	Em.	
	21	9 8 6 ·8	17 5 47 ·2	Em.	
II.	2	13 36 36 ·4	20 20 6 ·3	Em.	
	6	2 53 59 ·2	9 51 29 ·8	Em.	
	9	16 11 18 ·5	23 22 49 ·8	Em.	
	13	5 28 34 ·7	12 54 6 ·6	Em.	
	16	18 45 51 ·1	2 25 23 ·6	Em.	
	20	8 3 2 ·4	15 56 35 ·6	Em.	
III.	5	15 35 31 ·9	22 31 11 ·0	Em.	
	12	19 34 26 ·8	2 58 21 ·0	Em.	
	19	23 33 8 ·5	7 25 18 ·0	Em.	
IV.	11	0 26 3 ·4	7 42 52 ·5	Im.	
	11	5 8 43 ·1	12 26 18 ·6	Em.	

THE ECLIPSES OF THE SATELLITES OF JUPITER

are not visible

from the 22nd day of July until the 21st day of September,

JUPITER being too near to the SUN.

CONFIGURATIONS OF THE SATELLITES OF JUPITEI

THE SATELLITES OF JUPITER

are not visible this Month,

JUPITER being too near to the SUN.

ECLIPSES OF THE SATELLITES OF JUPITER.

The ECLIPSES of the SATELLITES of JUPITER

are not visible this Month,

JUPITER being too near to the SUN.

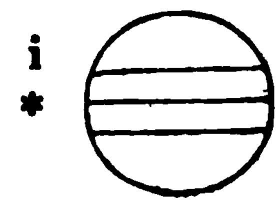
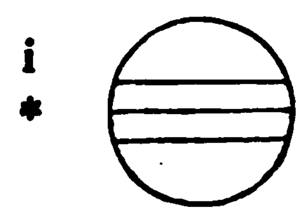
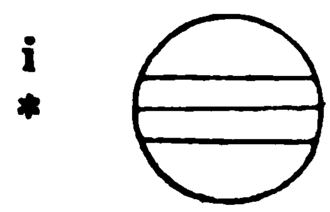
APPROXIMATE SIDEREAL TIMES
OF THE
OCCULTATIONS OF JUPITER'S SATELLITES BY JUPITER,
AND OF THE
TRANSITS OF THE SATELLITES AND THEIR SHADOWS OVER THE DISC OF THE PLANET.

THE SATELLITES OF JUPITER

are not visible this Month,

JUPITER being too near to the SUN.

ECLIPSES OF THE SATELLITES OF JUPITER.*

.LITE.	Day of the Month.	Mean Time.	Sidereal Time.	PHASE as seen in an inverting Telescope.	
		h m s	h m s		
.	21	5 33 32 ·8	17 35 4 ·4	Im.	
	23	0 2 1 ·5	12 10 31 ·8	Im.	
	24	18 30 34 ·4	6 46 3 ·4	Im.	
	26	12 59 2 ·1	1 21 29 ·6	Im.	
	28	7 27 34 ·3	19 57 0 ·5	Im.	
	30	1 56 1 ·9	14 32 26 ·7	Im.	
[.	22	4 18 49 ·7	16 24 5 ·6	Im.	
	25	17 35 32 ·9	5 54 49 ·3	Im.	
	29	6 52 15 ·1	19 25 32 ·1	Im.	
I.	22	7 49 54 ·2	19 55 44 ·7	Im.	
	29	11 48 31 ·1	0 22 36 ·7	Im.	

* The Satellites are not visible until the 21st day of this Month, Jupiter being too near to the Sun.

$f = -3''\cdot 43$; $g = +7''\cdot 67$; $G = 258^\circ\ 46'$; $h = +19''\cdot 48$; $H = 315^\circ\ 24'$; $i = -5''\cdot 93$

α (*in time*) converted $= 79\ \ 6$ . . . . $79\ \ 6$

$G + \alpha = 337\ \ 52$ $\qquad\qquad$ $H + \alpha = 34\ \ 30$

	Logarithms.	Nat. Nos.		Logarithms.	Nat.
f		$-3''\cdot 43$			
g	$+0\cdot 8848$			$+0\cdot 8848$	
$\sin(G+\alpha)$	$-9\cdot 5761$		cos	$+9\cdot 9668$	
$\tan \delta$	$+9\cdot 0357$				
				$+0\cdot 8516$	$+7''\cdot$[illegible]
	$-9\cdot 4966$	$-0\cdot 31$			
h	$+1\cdot 2896$			$+1\cdot 2896$	
$\sin(H+\alpha)$	$+9\cdot 7531$		cos	$+9\cdot 9160$	
$\sec \delta$	$+0\cdot 0025$		sin	$+9\cdot 0331$	
	$+1\cdot 0452$	$+11\cdot 10$		$+0\cdot 2387$	$+1\cdot$[illegible]
	$\Delta\alpha$ (*in arc*) $=$	$+7\cdot 36$	i	$-0\cdot 7731$	
	$\Delta\alpha$ (*in time*) $=$	$+0^s\cdot 49$	$\cos \delta$	$+9\cdot 9975$	
				$-0\cdot 7706$	$-5\cdot$[illegible]
				$\Delta\delta =$	$+2\cdot$[illegible]

Hence the App. Right Ascens. of γ Orionis $= 5^h\ 16^m\ 23^s\cdot 47 + 0^s\cdot 49 = 5^h\ 16^m\ 23^s\cdot 96$

And the Apparent Declination . . . . . . $= +6^\circ\ 11'\ 43''\cdot 86 + 2''\cdot 94 = +6^\circ\ 11'\ 46''\cdot 80$

been actually observed. The traveller has thus an opportunity of rendering his observations immediately available for determining his longitude with considerable accuracy.

The Right Ascension of the Moon's bright limb is given for the lower as well as the upper Culmination, *l. c.* being put to denote the Lower Culmination, and *u. c.* the Upper Culmination; the former is also distinguished by Italic letters; the Roman numerals indicate the limb of the Moon with reference to its transit over the meridian. The Right Ascension of the Moon's bright limb is given *for every day*, with a view to the more accurate determination of its variation when required. The Moon's age at the time of her upper transit, to the nearest tenth of a day, is inserted in a parenthesis in the column containing the Magnitudes of the Stars.

The Declinations are given to the nearest minute, and are useful for pointing the instrument to the object.

The numbers in the column Var. of ☾'s R. A. in 1 hour of Long. represent the Variation in Right Ascension of the Moon's Limb during the interval of her transit over two meridians, equidistant from that of Greenwich, and *one* hour distant from each other. They have been deduced from the Right Ascensions of the bright *limb*, and therefore include the effect produced by the change of the semidiameter. They serve to determine the Longitude where the difference of meridians is not very great; but where this difference is considerable, and extreme accuracy is wanted, the variation in Right Ascension should be used which corresponds to the middle of the interval between the observations, which may be readily obtained by interpolation. They also serve to determine the Right Ascension of the bright limb at its transit over any other meridian. Thus: Multiply the difference of longitude between Greenwich and the given meridian, by the variation; and, according as the given meridian is east or west of Greenwich, subtract or add the product to the Right Ascension at Greenwich; the result will be the Right Ascension of the bright limb at transit over the proposed meridian. *Example:* On May 19, 1837, the Right Ascension of the Moon's first limb is $15^h\ 24^m\ 51^s{\cdot}86$, at its upper transit at Greenwich, and the variation for one hour of longitude is $147^s{\cdot}21$: Required the Right Ascension of the limb at its upper transit at Paris. Paris is $9^m\ 21^s{\cdot}5$, or $0^h{\cdot}156$, East of Greenwich; therefore, multiplying $147^s{\cdot}21$ by $0{\cdot}156$, and subtracting the product $22^s{\cdot}96$ from $15^h\ 24^m\ 51^s{\cdot}86$, we have $15^h\ 24^m\ 28^s{\cdot}90$, for the Right Ascension at Paris.

Where an asterisk is placed opposite to a Star's name, it is intended to denote that the Star is favourably situated for observing its Declination along with that of the Moon in both hemispheres, with a view to the accurate determination of the Moon's Parallax.

The numbers in the column entitled Sid. Time of ☾'s Sem. pass. mer., express the Sidereal intervals which the Moon's Semidiameter, at the time of transit at Greenwich, takes in passing the meridian, and therefore serve to determine the Transit of the centre from an observed Transit of either limb.

Occultations. (Pages 452 to 453.)

These pages contain a list of the Planets and Fixed Stars to the sixth magnitude inclusive, the Occultations of which by the Moon will happen when the objects are above the horizon of Greenwich, together with the Sidereal and Mean Times of the Immersions and Emersions, and the points on the circumference of the Moon's image, where the Star, viewed with a telescope that inverts, will disappear and reappear. By

APPENDIX

TO THE

NAUTICAL ALMANAC FOR 1837.

or, as the quantity under the differential sign on the first side is $= \frac{1}{a}$,

$$-\frac{1}{a^2} \cdot \frac{da}{dt} = 2\,A\,\frac{dx}{dt} + 2\,B\,\frac{dy}{dt} + 2\,C\,\frac{dz}{dt}$$

$$\text{whence } \frac{da}{dt} = -\,2\,a^2 \left(A\,\frac{dx}{dt} + B\,\frac{dy}{dt} + C\,\frac{dz}{dt}\right)$$

from which the variation of the semi-major axis of the orbit is found.

(8) Now the longitude of m from the node is $\theta - \nu$ (θ being measure mentioned in (4)), and the co-ordinates of m parallel to the line of nodes, per dicular to the line of nodes in the plane of the ecliptic, and perpendicular to ecliptic, are therefore $r \cos(\theta-\nu)$, $r \sin(\theta-\nu) \cos i$, and $r \sin(\theta-\nu)$ s From these we readily obtain

$$x = r\,\{\cos(\theta-\nu)\cos\nu - \sin(\theta-\nu)\cos i \sin\nu\}$$
$$y = r\,\{\sin(\theta-\nu)\cos i \cos\nu + \cos(\theta-\nu)\sin\nu\}$$
$$z = r \sin(\theta-\nu)\sin i$$

and hence

$$(12) \quad \frac{\quad}{dt} = \cos^2\nu \frac{\quad}{dt}$$

$$= \cos^2\nu \frac{d}{dt} \cdot \frac{h \sin i \sin \nu}{h \sin i \cos \nu}$$

$$= \frac{1}{h^2 \sin^2 i} \left\{ h \sin i \cos \nu \frac{d}{dt} (h \sin i \sin \nu) - h \sin i \sin \nu \frac{d}{dt} (h \sin i \right.$$

$$= \frac{1}{h \sin i} \left\{ \cos \nu \frac{d}{dt} (h \sin i \sin \nu) - \sin \nu \frac{d}{dt} (h \sin i \cos \nu) \right\}$$

$$= \frac{\mu}{h \sin i} \left\{ \cos \nu (Bz - Cy) - \sin \nu (Az - Cx) \right\}$$

$$= \frac{\mu}{h \sin i} \left\{ A(-z \sin \nu) + B(z \cos \nu) + C(x \sin \nu - y \cos \nu) \right\}$$

If we substitute for x and y the values in (8), we find

$$x \sin \nu - y \cos \nu = -r \sin(\theta - \nu) \cos i = -z \cot i$$

Therefore $\dfrac{d\nu}{dt} = \dfrac{\mu z}{h \sin^2 i} \left\{ -A \sin \nu \sin i + B \cos \nu \sin i - C \cos i \right\}$

Let $-A \sin \nu \sin i + B \cos \nu \sin i - C \cos i = C'$

Substituting the values of x, y, and z, from (8)

$$\frac{1}{h}\cdot\frac{dh}{dt}=\frac{anr}{\sqrt{(1-e^2)}}\left\{A\left(\sin(\theta-\nu)\cos\nu+\cos(\theta-\nu)\cos i\sin\nu\right)\right.$$

$$\left.+B\left(-\cos(\theta-\nu)\cos i\cos\nu+\sin(\theta-\nu)\sin\nu\right)-C\sin i\cos(\theta-\nu)\right\}$$

$$=\frac{an}{\sqrt{(1-e^2)}}\,r\,B'$$

And as $h=\sqrt{\mu}\,\sqrt{\{a(1-e^2)\}}$, we get

$$\frac{1}{h}\cdot\frac{dh}{dt}=\frac{1}{2a}\cdot\frac{da}{dt}-\frac{e}{1-e^2}\cdot\frac{de}{dt}$$

$$\text{or }\frac{an}{\sqrt{(1-e^2)}}\,r\,B'=-\frac{na^2e}{\sqrt{(1-e^2)}}A'\cdot\frac{\sin(\theta-\varpi)}{r}$$

$$+na^2\sqrt{(1-e^2)}\cdot\frac{B'}{r}-\frac{e}{1-e^2}\cdot\frac{de}{dt}$$

whence

$$\frac{e}{1-e^2}\cdot\frac{de}{dt}=-\frac{na^2e}{\sqrt{(1-e^2)}}A'\frac{\sin(\theta-\varpi)}{r}+\frac{na}{\sqrt{(1-e^2)}}\left(a^2(1-e^2)-r^2\right)\frac{B'}{r}$$

$$\text{and }\frac{de}{dt}=-na^2\sqrt{(1-e^2)}\frac{\sin(\theta-\varpi)}{r}A'+\frac{na\sqrt{(1-e^2)}}{e}\left(a^2(1-e^2)-r^2\right)\frac{B'}{r}$$

(15) Now $\frac{d(\log r)}{dt}$ is found by differentiating

$$\log r=\log a+\log(1-e^2)-\log\{1+e\cos(\theta-\varpi)\},$$

which represents the correct value of $\log r$, because by (2) and (4) the same expressions are to be taken to represent the place of m (using the elements of the instantaneous ellipse), as those which are employed in undisturbed elliptic motion (using the elements of the permanent ellipse). Still it is to be borne in mind that the elements vary from one instant to another; and therefore their variation must be taken into account in forming $\frac{d(\log r)}{dt}$. Thus we have for $\frac{d(\log r)}{dt}$ the rigorous expression

$$\frac{d(\log a)}{dt}+\frac{d\{\log(1-e^2)\}}{dt}-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{de}\cdot\frac{de}{dt}$$

$$-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{d\varpi}\cdot\frac{d\varpi}{dt}-\frac{d\{\log(1+e\cos(\theta-\varpi))\}}{d\theta}\cdot\frac{d\theta}{dt}.$$

This expression, it is evident, has been obtained merely by considering that the *place* of m is always represented truly by the elliptic formulæ applied to the variable elements. But by (2) the *motion* of m is also to be represented truly by the elliptic formulæ for motion applied to the variable elements: and therefore generally the first differential coefficient with respect to t, of r or θ, or of any function of r or θ, must be

the epoch, the element by which the body's place in the orbit is fou purpose let us consider that θ, the longitude at the time t, depends upon a, e, ϖ and ϵ, inasmuch as it is expressed by the series

$$nt + \epsilon + (2e + \&c.) \sin (nt + \epsilon - \varpi) + \&c.$$

where $n = \sqrt{\mu}\, a^{-\frac{3}{2}}$. The reasoning of (15), applied to this instance we must have

$$\frac{d\theta}{da} \cdot \frac{da}{dt} + \frac{d\theta}{de} \cdot \frac{de}{dt} + \frac{d\theta}{d\varpi} \cdot \frac{d\varpi}{dt} + \frac{d\theta}{d\epsilon} \cdot \frac{d\epsilon}{dt} = 0$$

The values of $\frac{da}{dt}$, $\frac{de}{dt}$, $\frac{d\varpi}{dt}$, have been found; we have therefore only the values of $\frac{d\theta}{da}$, $\frac{d\theta}{de}$, $\frac{d\theta}{d\varpi}$, and $\frac{d\theta}{d\epsilon}$, and then the equation above will giv

(18) For $\frac{d\theta}{da}$. Whatever be the definition of a differential coefficie tical rule for finding it is this: give to a the increment δa, and le elements and the time unaltered; find the increment $\delta\theta$ which this caus then take the value to which $\frac{\delta\theta}{\delta a}$ approaches when δa is made indefi Now θ is found in terms of t by integrating this expression

$$\frac{dt}{d\theta} = \frac{r^2}{h}$$

$$= \frac{a^2(1-e^2)^2}{\{1+e \cos (\theta-\varpi)\}^2} \times \frac{1}{\sqrt{\mu}\sqrt{\{a(1-e^2)\}}}$$

If now we put $a+\delta a$ instead of a, $\dfrac{dt}{d\theta}$ becomes

$$\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\,\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}+\text{higher powers of }\delta a:$$

and integrating,

$$t=\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}f(\theta)+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\Big(f(\theta)-\text{constant}\Big)+\text{higher powers of }\delta a.$$

Putting $\theta+\delta\theta$ in the place of θ,

$$t=\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}f(\theta)+\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{d.f(\theta)}{d\theta}\,\delta\theta+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\Big(f(\theta)-\text{constant}\Big)$$
$$+\text{higher powers and combinations of }\delta a\text{ and }\delta\theta.$$

Making this value of t equal to the former,

$$\frac{a^{\frac{3}{2}}}{\sqrt{\mu}}\cdot\frac{d.f(\theta)}{d\theta}\,\delta\theta+\frac{3}{2}\cdot\frac{a^{\frac{1}{2}}}{\sqrt{\mu}}\,\delta a\Big(f(\theta)-\text{constant}\Big)+\&\text{c.}=0\,;$$

whence the limit of the value of $\dfrac{\delta\theta}{\delta a}$ is

$$=-\frac{3}{2a}\cdot\frac{f(\theta)-\text{constant}}{\dfrac{df(\theta)}{d\theta}}$$

$$=-\frac{3}{2a}\cdot\frac{\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}\Big(f(\theta)-\text{constant.}\Big)$$

To determine the constant, it is to be remarked, that from the beginning we have assumed that in the instantaneous ellipse, whatever changes it may undergo, $nt+\epsilon$ is to represent the mean longitude: and therefore the variation of n (and consequently the variation of a on which n depends) must have t for a factor; and therefore its effect in $nt+\epsilon$, and in θ which depends on $nt+\epsilon$, must vanish when $t=0$, or when $f(\theta)=0$. From this we find,

$$\text{constant}=0:$$

$$\frac{d\theta}{da}\text{ or the limit of }\frac{\delta\theta}{\delta a}=-\frac{3}{2a}\cdot\frac{\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}\cdot\frac{\sqrt{\mu}}{a^{\frac{3}{2}}}\,t$$

$$=-\frac{3}{2}\,na\sqrt{(1-e^2)}\,\frac{\{1+e\cos(\theta-\varpi)\}^2}{a^2(1-e^2)^2}\,t$$

$$=-\frac{3}{2}\,na\sqrt{(1-e^2)}\,\frac{t}{r^2}.$$

The reader's attention is particularly invited to the circumstance, that in this investigation distinct reference is made to the assumption, that in the instantaneous ellipse the mean longitude is found by adding to the epoch the mean motion corresponding to that ellipse since $t=0$.

$$+ \text{ higher powers and combinations of } \delta\theta \text{ and } \delta e + \text{constant} \times \delta e.$$

Supposing t, and all the elements except e unaltered, this value of t must be same as if θ and e had no variations, that is it must be $\frac{1}{n} f(\theta)$. Consequently

$$0 = \frac{1}{n} \cdot \frac{d.f(\theta)}{d\theta} \delta\theta - \frac{\sqrt{(1-e^2)}}{n} \cdot \frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2} \delta e$$

$$+ \text{ higher powers, \&c.} + \text{constant} \times \delta e$$

whence
$$\frac{d\theta}{de} = \frac{\sqrt{(1-e^2)}}{\frac{d.f(\theta)}{d\theta}} \cdot \frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{\{1+e\cos(\theta-\varpi)\}^2} + \text{constant} \times \frac{n}{\frac{d.f(\theta)}{d\theta}}$$

$$= \frac{\sin(\theta-\varpi)\{2+e\cos(\theta-\varpi)\}}{1-e^2} + \text{constant} \times \frac{n\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}.$$

To determine the constant we must observe that in the elliptic expressions the part of θ which depends on e is the equation of the centre; and that this is 0, its variation produced by a variation of e is 0, when $\theta - \varpi = 0$; and therefore must have $0 + \text{constant} \times \frac{n(1+e)^2}{(1-e^2)^{\frac{3}{2}}} = 0$,

hence

$$\text{constant} = 0,$$

and therefore

$$\frac{d\theta}{de} = \frac{\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}}{1-e^2}.$$

(20) For $\frac{d\theta}{d\varpi}$. As before, t is found by integrating (with respect to θ)

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}$$

when the longitude of perihelion is ϖ: and therefore when the longitude of perihelion is $\varpi+\delta\varpi$, t will be found by integrating

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} + \delta\varpi\,\frac{d}{d\varpi}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right) + \text{higher powers of } \delta\varpi.$$

From the manner in which θ and ϖ enter into the last term, it is evident that

$$\frac{d}{d\varpi}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right) = -\frac{d}{d\theta}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right);$$

and therefore t will now be found by integrating, with respect to θ,

$$\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} - \delta\varpi\,\frac{d}{d\theta}\left(\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\right)$$

or the value of t is

$$\frac{1}{n}f(\theta) - \delta\varpi\,\frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2} + \text{higher powers of } \delta\varpi + \text{constant} \times \delta\varpi.$$

Put $\theta+\delta\theta$ in the place of θ, as before; this becomes

$$\frac{1}{n}f(\theta) + \frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\,\delta\theta - \frac{1}{n}\cdot\frac{(1-e^2)^{\frac{3}{2}}}{\{1+e\cos(\theta-\varpi)\}^2}\,\delta\varpi$$

$$+ \text{constant} \times \delta\varpi$$

$$+ \text{higher powers and combinations of } \delta\theta \text{ and } \delta\varpi.$$

From this as in the former cases,

$$\frac{d\theta}{d\varpi} = 1 - \text{constant} \times \frac{n\{1+e\cos(\theta-\varpi)\}^2}{(1-e^2)^{\frac{3}{2}}}$$

$$= 1 - \text{constant} \times \frac{na^2\sqrt{(1-e^2)}}{r^2}.$$

To determine the constant, we have to observe that the only part for θ which depends on ϖ is the equation of the centre: an[illegible] [illegible]t

(22) Now substituting all the values in the equation of (17),

$$-\frac{3}{2} na\sqrt{(1-e^2)}\,\frac{t}{r^2}\times\left\{-2\frac{na^3e}{\sqrt{(1-e^2)}}\,A'\,\frac{\sin(\theta-\varpi)}{r}+2na^4\sqrt{(1-e^2)}\right.$$

$$+\frac{\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}}{1-e^2}\times\left\{-na^2\sqrt{(1-e^2)}\,\frac{\sin(\theta-\varpi)}{r}\,A'\right.$$

$$\left.+\frac{na\sqrt{(1-e^2)}}{e}\,\{a^2(1-e^2)-r^2\}\,\frac{B}{r}\right.$$

$$+\left(1-\frac{a^2\sqrt{(1-e^2)}}{r^2}\right)\times\left\{\frac{na^2\sqrt{(1-e^2)}}{e}\cdot\frac{\cos(\theta-\varpi)}{r}\,A'\right.$$

$$\left.+\frac{na}{e\sqrt{(1-e^2)}}\,r\sin(\theta-\varpi)\,\{2+e\cos(\theta-\varpi)\}\,B'\right\}$$

$$+\frac{a^2\sqrt{(1-e^2)}}{r^2}\cdot\frac{d\epsilon}{dt}=0$$

From which $\frac{d\epsilon}{dt}=\left\{-3\,\frac{n^2a^3e}{\sqrt{(1-e^2)}}\,t\,\frac{\sin(\theta-\varpi)}{r}+3na\right.$

$$\left.+\frac{na^2(1-e^2)}{e}\left(\frac{1}{\sqrt{(1-e^2)}}-1\right)\frac{\cos(\theta-\varpi)}{r}\right\}A'$$

$$A = \frac{m_1}{\mu}\left\{\frac{x-x_1}{\lambda_1^3} + \frac{x_1}{r_1^3}\right\} + \frac{m_2}{\mu}\left\{\frac{x-x_2}{\lambda_2^3} + \frac{x_2}{r_2^3}\right\} + \&c.$$

$$B = \frac{m_1}{\mu}\left\{\frac{y-y_1}{\lambda_1^3} + \frac{y_1}{r_1^3}\right\} + \frac{m_2}{\mu}\left\{\frac{y-y_2}{\lambda_2^3} + \frac{y_2}{r_2^3}\right\} + \&c.$$

$$C = \frac{m_1}{\mu}\left\{\frac{z-z_1}{\lambda_1^3} + \frac{z_1}{r_1^3}\right\} + \frac{m_2}{\mu}\left\{\frac{z-z_2}{\lambda_2^3} + \frac{z_2}{r_2^3}\right\} + \&c.$$

(38) Find the angles ψ and χ, where $\tan\psi = \tan\nu\cos i$, and $\tan\chi = \cot$ (ψ and χ will therefore be constants): and calculate for every middle day the ing expressions:—

$$A' = Ax + By + Cz$$

$$B' = A\frac{\cos\nu}{\cos\psi}\sin(\theta-\nu+\psi) + B\frac{\sin\nu}{\cos\chi}\sin(\theta-\nu-\chi) - C\sin i\cos(\theta-\nu)$$

$$C' = -A\sin\nu\sin i + B\cos\nu\sin i - C\cos i.$$

(39) Find the angle ϕ, such that $\sin\phi = e$ (ϕ is therefore constant), and cal for each middle day the following expressions (where p, as before mentioned, ordinal number of the interval)

The true longitude, calculated with the elements thus further corrected and the mean sidereal motion, for any considerable interval before and after the day for which the elements are computed, must also be affected with precession proportional to that interval. If, however, in the process of calculating true longitudes, the motion of precession be added to the mean sidereal motion, and if the same motion of precession be applied to the longitudes of the node and perihelion (neglecting for short times the effect of change of obliquity) it will not be necessary to take account of precession afterwards.

G. B. AIRY.

OBSERVATORY, CAMBRIDGE,
Dec. 3, 1834.

✻'s corrᵈ. R. A.	-	10ʰ 21ᵐ 13ˢ·83
☽'s R. A.	- - - -	10 20 58 ·47
diff.	- - - - - - -	0 15 ·36
common log.	- - - - - -	1 ·1864
com. log. h. m. ☽'s R. A.	-	2 ·0896
Remainder	- -	0 ·9032
const. log.	- - - - - - -	0 ·4771
corrⁿ. - - - 0ʰ 7ᵐ 29ˢ·9	P. L.	1 ·3803
Time assumed 11 0 0		
Time corrᵈ. - 11 7 29 ·9		

	- - - - - - - - - - - - - -	0 ·903[illegible]
P. L. h. m. ☽'s dec.	- - -	1 ·187[illegible]
corrⁿ. - - S. 0° 1′ 27″·7	P. L.	[illegible]
☽'s dec. - - N. 15 47 11 ·0		
☽'s dec. corrᵈ. N. 15 45 43 ·3		

com. log. h. m. ☽'s R. A.	- - - -	2 ·0896
cos. ☽'s corrᵈ. dec.	- - - - - - -	9 ·9834
sum (S_1)	- - - - - - - - - - - -	2 ·0730
P. L. h. m. ☽'s dec.	- - - - - - -	1 ·1874
const. log.	- - - - - - - - - - - -	7 ·1427
1st Orb. incl. - - N. 21° 34′	cot.	0 ·4031
2nd Orb. incl. - - S. 61 9		
sum - - - - - S. 39 35		

✻'s corrᵈ. dec.	- - -	N. 15° 37′ 47″·[illegible]
☽'s ———	- -	N. 15 45 43 ·[illegible]
diff. (✻ S. of ☽)	- -	7 55 ·[illegible]
P. L.	- - - - - - - -	1 ·3563
const. log.	- - - - - -	9 ·4354
sec.	- - - - - - - - -	0 ·0315
		0 ·8232
P. L. ☽'s hor. par.	- -	0 ·5068
sec.	- - - - - - - - -	0 ·3164
cosec.	- - - - - - - -	0 ·1957
sum (S_2)	- - - - - - -	2 ·0730
P. L. ☽'s hor. par.	- -	0 ·5068
const. log.	- - - - - - -	8 ·1844
P. L.	- - - - - - - - -	0 ·9599

corrⁿ.		0ʰ 19ᵐ 44ˢ·5
Time corrᵈ.		11 7 29 ·9
Greenwʰ mean time	- -	10 47 45 ·4
Equation of time	- - -	6 31 ·0
Greenwʰ app. time	- -	10 41 14 ·4
Observed ———	- -	10 39 22 ·4
Longitude	- - - - - -	1 52 ·0 W.

P.S.—The principle of reversing the effect of the relative horizontal parallax on the position of the Sun, instead of using the actual effect on the position of the Moon, may be advantageously employed in the direct calculation of an Eclipse for a particular place. It will only be necessary to use the parallaxes for the Sun viewed as an apparent position, and to diminish the semidiameter by the amount derived from the table on page 175. Thus, it appears, at the beginning of the Eclipse, for instance, that the contact may be mathematically tested in two ways. First, we may apply the actual effects of the parallax to the true position of the Moon, then augment her semidiameter, and thus establish a contact of the limbs. But, if we reverse the operation, and consider the Sun to be an apparent body under the influence of the relative parallax, then clearing it from this supposed influence by reversing the parallax, and

diminishing the semidiameter, a contact will similarly be established with the true limb of the Moon; and this principle, in its application to solar eclipses, possesses an advantage similar to that derived in the case of an occultation, by considering the Star as an apparent place. (See *Appendix to Nautical Almanac* for 1836, page 125*.)

The formulæ, Nos. 2, 3, 4, and 5, pages 130 and 131 may, according to this method, be supplied by the following:

2.

$$P' = \rho\,(P - \pi) \qquad m = P' \cos l$$

$$Q_1 = [9{\cdot}4180] \qquad Q_2 = [9{\cdot}4180]\; m \sin \delta$$

$$s = [9{\cdot}43537]\; P'$$

3.

$$k = \frac{m}{\cos D}$$

$$\Delta\, h \text{ in minutes} = [7{\cdot}92082]\; k \sin h$$

$$(h) = h - \Delta\, h$$

$$\tan \theta = \cos\,(h) \cot l \qquad G = \cos\,(h) \cos l$$

$$\tan M = \frac{\sin \theta}{\cos\,(\theta + \delta)} \tan\,(h) \qquad \tan \epsilon = \tan\,(\theta + \delta) \cos M$$

$$B = \cos M \cos \epsilon$$

$$\text{check - - - } \frac{\sin \theta}{\cos\,(\theta + \delta)} = \frac{G}{B}$$

$$\Delta\,\delta = B.\,P'$$

$$\sigma_0 = \sigma - \text{diminution for } \epsilon$$

$$\text{For} \left\{ \begin{array}{l} \text{partial} \\ \text{total or annular} \end{array} \right\} \text{phase, } \Delta' = \left\{ \begin{array}{l} s + \sigma_0 \\ s - \sigma_0 \end{array} \right.$$

4.

$$k_0 = \frac{m}{\cos \delta_0} \qquad \Delta\,\alpha = k_0 \sin h$$

$$\Delta\,\alpha_1 = Q_1\, k_0 \cos h \qquad \Delta\,\delta_1 = Q_2 \sin\,(h)$$

5.

$$\delta_0 = \delta + \Delta\,\delta \qquad \alpha' = \alpha - \Delta\,\alpha$$

$$y = (\alpha - \Delta\,\alpha) \cos D \qquad y_1 = (\alpha_1 - \Delta\,\alpha_1) \cos D$$

$$x = (D + \alpha' \text{ corr.}) - \delta_{\sigma} \qquad x_1 = D_1 - \Delta\,\delta_1$$

* This was inadvertently ascribed to Carlini; Professor Henderson, by w appeared upon this very point, in the Quarterly Journal for 1828, page 411, i method has been long in practice, and that it was employed at an early period by